Possible large branching fraction of ψ'' decays to charmless final states

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The branching fraction of the ψ'' decays into charmless final states is estimated in the S- and D-wave charmonia mixing scheme. With the information of the hadronic decays of J/ψ and ψ', it is found that a large branching fraction, up to 13% of the total ψ'' decays may go to charmless final states. The experimental search for these decays is also discussed.

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I. INTRODUCTION

As more and more data are collected at $\psi(3770)$ (shortened as ψ''), 27.7 pb^{-1} by BES-II, 55 pb^{-1} by CLEOc, and a few fb^{-1} is in progress by CLEOc, the study of the non-$D\overline{D}$ decays of ψ'' gets renaissance recently.

Due to its mass coincides with the $D\overline{D}$ mass threshold, ψ'' is believed to decay predominantly to $D\overline{D}$, and the non-$D\overline{D}$ decays including transitions between charmonium states and decays into light hadrons are expected to be small. The first exclusive non-$D\overline{D}$ decay mode of ψ'' was searched for using the Mark-III data sample [1], where evidence of $\psi'' \to J/\psi\pi^+\pi^-$ was observed; afterwards theoretical calculation of its decay width was published [2]. This decay mode was searched for recently by BES and a signal of 3.1σ was observed [3], which is in marginally agreement with the upper limit set by CLEOc [4]. The other searches of the non-$D\overline{D}$ decays of ψ'' were all reported in either doctoral theses [1, 5] based on the Mark-III data or conference talk [6] based on the BES-II data, and no statistically significant results were given. This may indicate that the data samples utilized are still not large enough to search for these channels of small branching fractions. Now with much larger ψ'' samples, measurements of the cross section $\sigma(e^+e^- \to \psi'' \to D\overline{D}) \equiv \sigma(D\overline{D})$ have recently been reported by BES-II [6] and CLEOc [7] collaborations using either single-tag or double-tag method. Comparing with the measured $\sigma(e^+e^- \to \psi'' \to \text{anything})$, the difference can be up to 1.4 nb (about 18% of the total cross section of the ψ'' production in e^+e^- collision [8]), which indicates the existence of substantial non-$D\overline{D}$ decays for ψ''. However, there is limited theoretical work on this aspect. In Ref. [8], it is estimated that at most 600 keV ($\sim 2.5\%$) of the ψ'' total width of (23.6 ± 2.7) MeV is due to the radiative transition, and perhaps as much as another 100 keV ($\sim 0.4\%$) is due to the hadronic transition to $J/\psi\pi\pi$. All these together are far from accounting for a deficit of 18% of the total ψ'' width.

In this paper, we concentrate on the charmless decays of ψ''. By charmless decay, we exclude those decay modes with either open or hidden charm. We shall estimate the partial width of the charmless decay of ψ'' in the $2S$-$1D$ charmonia mixing scenario, under the assumption that the perturbative QCD (pQCD) "12% rule" holds between pure $1S$ and $2S$ charmonium states. The mixing between $2S$ and $1D$ states of charmonium was originally proposed to explain the measured large Γ_{ee} of ψ''. It is suggested [9] that the mass eigenstates $\psi(3686)$ (shortened as ψ') and ψ'' are the mixtures of the $2S$- and $1D$-wave of charmonia, namely $\psi(2^3S_1)$ and $\psi(1^3D_1)$ states. This was used by Rosner [10] in explaining the "$\rho\pi$ puzzle" in ψ' and J/ψ decays. He suggested that the mixing of $\psi(2^3S_1)$ and $\psi(1^3D_1)$ states is in such a way which leads to almost complete cancellation of the decay amplitude of $\psi' \to \rho\pi$, and the missing $\rho\pi$ decay mode of ψ' shows up instead as enhanced decay mode of ψ''. This idea was then applied to solve the enhanced decay of $\psi' \to K^0_S K^0_L$ relative to $J/\psi \to K^0_S K^0_L$, and a prediction of the branching fraction of $\psi'' \to K^0_S K^0_L$ was given [11]. In principle, if this scenario is correct, it can be generalized and relate the partial widths of each individual mode in J/ψ, ψ' and ψ'' decays. Therefore by virtue of the branching fractions of J/ψ and ψ' decays, we are able to estimate the corresponding decay branching fraction of ψ''. This will be tested by the upcoming large ψ'' data sample from CLEOc.

In the following parts of the paper, we begin our study with a general review of the 12% rule between ψ' and J/ψ decays, followed by an introduction of the $2S$- and $1D$-wave charmonia mixing scheme. Then we conduct the calculation on the possible charmless decay width of ψ'' with the available information from J/ψ and ψ' decays. We show that the scenario and currently available information on J/ψ and ψ' decays can accommodate a partial width of 3.0 MeV charmless decays of ψ''. Finally we turn to the experimental searches for these decays. Both inclusive and exclusive methods are examined.

II. 12% RULE AND $2S$-$1D$ MIXING

From the pQCD, it is expected that both J/ψ and ψ' decaying into light hadrons are dominated by the annihilation of $c\bar{c}$ into three gluons, with widths proportional to the square of the wave function at the origin $|\Psi(0)|^2$ [12].

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This yields the pQCD 12% rule, that is

$$Q_h = \frac{\mathcal{B}_{\psi'\to h}}{\mathcal{B}_{J/\psi\to h}} = \frac{\mathcal{B}_{\psi'\to e^+e^-}}{\mathcal{B}_{J/\psi\to e^+e^-}} \approx 12.7\%. \qquad (1)$$

The violation of the above rule was first observed in $\rho\pi$ and $K^{*+}K^- + c.c.$ modes by Mark-II [13]. Since then BES-I, BES-II and CLEOc have measured many two-body decay modes of ψ' [14–22]. Among them, some obey the rule , like baryon-antibaryon ($B\overline{B}$) modes, while others are either suppressed in ψ' decays, like vector-pseudoscalar (VP) and vector-tensor (VT) modes, or enhanced, like $K_S^0K_L^0$. There have been many theoretical efforts trying to solve the puzzle [10, 23]. Among them, the $2S$-$1D$ charmonia mixing scenario [10] predicts with little uncertainty $\mathcal{B}(\psi''\to\rho\pi)$ which agrees with experimental data [1, 24].

In S- and D-wave charmonia mixing scheme, the mass eigenstates ψ' and ψ'' are the mixtures of the S- and D-wave charmonia, namely

$$|\psi'\rangle = |2^3S_1\rangle\cos\theta - |1^3D_1\rangle\sin\theta\ ,$$
$$|\psi''\rangle = |2^3S_1\rangle\sin\theta + |1^3D_1\rangle\cos\theta\ ,$$

where θ is the mixing angle between pure $\psi(2^3S_1)$ and $\psi(1^3D_1)$ states and is fitted from the leptonic widths of ψ'' and ψ' to be either $(-27\pm2)^\circ$ or $(12\pm2)^\circ$ [10]. The latter value of θ is consistent with the coupled channel estimates [9, 25] as well as the ratio between ψ' and ψ'' partial widths to $J/\psi\pi^+\pi^-$ [2]. Hereafter, the calculations and discussions in this paper are solely for the mixing angle $\theta = 12^\circ$ [26].

As in the discussion of Ref. [10], since both hadronic and leptonic decay rates are proportional to the square of the wave function at the origin, it is expected that if ψ' is a pure $\psi(2^3S_1)$ state, then for any hadronic final states f,

$$\Gamma(\psi'\to f) = \Gamma(J/\psi\to f)\frac{\Gamma(\psi'\to e^+e^-)}{\Gamma(J/\psi\to e^+e^-)}\ . \qquad (2)$$

The electronic partial width of J/ψ is expressed in potential model by [27]

$$\Gamma(J/\psi\to e^+e^-) = \frac{4\alpha^2e_c^2}{M_{J/\psi}^2}\left|R_{1S}(0)\right|^2,$$

with α the QED fine structure constant, $e_c = 2/3$, $M_{J/\psi}$ the J/ψ mass and $R_{1S}(0)$ the radial 1^3S_1 wave function at the origin.

Since ψ' is not a pure $\psi(2^3S_1)$ state, its electronic partial width is expressed as [10]

$$\Gamma(\psi'\to e^+e^-) = \frac{4\alpha^2e_c^2}{M_{\psi'}^2} \times\left|\cos\theta R_{2S}(0) - \frac{5}{2\sqrt{2}m_c^2}\sin\theta R''_{1D}(0)\right|^2,$$

with $M_{\psi'}$ the ψ' mass, m_c the c-quark mass, $R_{2S}(0)$ the radial 2^3S_1 wave function at the origin and $R''_{1D}(0)$ the second derivative of the radial 1^3D_1 wave function at the origin. In the calculations in this paper, we take $R_{2S}(0) = 0.734$ GeV$^{3/2}$ and $5R''_{1D}(0)/(2\sqrt{2}m_c^2) = 0.095$ GeV$^{3/2}$ from Refs. [10, 28].

If Eq. (2) holds for a pure $2S$ state, $\psi''\to f$, $\psi'\to f$ and $J/\psi\to f$ partial widths are to be [11]

$$\Gamma(\psi''\to f) = \frac{C_f}{M_{\psi''}^2}\left|\sin\theta R_{2S}(0) + \eta\cos\theta\right|^2,$$
$$\Gamma(\psi'\to f) = \frac{C_f}{M_{\psi'}^2}\left|\cos\theta R_{2S}(0) - \eta\sin\theta\right|^2,$$
$$\Gamma(J/\psi\to f) = \frac{C_f}{M_{J/\psi}^2}\left|R_{1S}(0)\right|^2\ , \qquad (3)$$

where C_f is a common factor for the final state f, $M_{\psi''}$ the ψ'' mass, and $\eta = |\eta|e^{i\phi}$ is a complex parameter with ϕ being the relative phase between $\langle f|1^3D_1\rangle$ and $\langle f|2^3S_1\rangle$.

From Eq. (3), it is obvious that with $\Gamma(J/\psi\to f)$ and $\Gamma(\psi'\to f)$ known, two of the three parameters, C_f and complex η, can be fixed, thus can be used to predict $\Gamma(\psi''\to f)$ with only one unknown parameter, say, the phase of η. Thus the S- and D-wave mixing scenario provides a mathematical scheme to calculate the partial width of ψ'' decay to any exclusive final state, with its measured partial widths in J/ψ and ψ' decays.

However, the current information concerning the ψ' decay is extremely limited, which prevents us from estimating Q_h values for most exclusive decay modes. Table I lists some hadronic final states which are measured both in J/ψ and ψ' decays, together with the calculated Q_h defined in Eq. (1). Summing up all the channels in Table I makes less than 2% of the ψ' decay through ggg annihilation.

From Table I, we notice that compared with the 12% rule, the ψ' decays to

1. the pseudoscalar-pseudoscalar (PP) mode $K_S^0K_L^0$ is enhanced;
2. the VP and VT modes are suppressed;
3. most of the $B\overline{B}$ modes are consistent with it.

The summed branching fractions and Q_h values for these three categories of decay modes are evaluated and also listed in Table I. In estimating the charmless decays of ψ'', we shall discuss these three different cases separately.

Since the experimental information on the exclusive decays of ψ' is rather limited, we turn to inclusive branching fractions of J/ψ and ψ' hadronic decays as an alternative. The estimation is based on the assumption that the decays of J/ψ and ψ' in the lowest order of QCD are classified into hadronic decays (ggg), electromagnetic decays (γ^*), radiative decays into light hadrons (γgg), and transition to lower mass charmonium states ($c\overline{c}X$) [31, 32]. Thus, using the relation $\mathcal{B}(ggg)+\mathcal{B}(\gamma gg)+\mathcal{B}(\gamma^*)+\mathcal{B}(c\overline{c}X) = 1$, one can derive

TABLE I: Branching fractions and Q_h values for some J/ψ and $\psi(2S)$ decay channels.

Modes	Channels	$\mathcal{B}_{J/\psi}(10^{-3})$	$\mathcal{B}_{\psi(2S)}(10^{-4})$	Q_h (%)	Ref.
0^-0^-	$\pi^+\pi^-$	0.147 ± 0.023	0.8 ± 0.5	54 ± 35	[29]
	K^+K^-	0.237 ± 0.031	1.0 ± 0.7	42 ± 30	[29]
	$K_S^0K_L^0$	0.182 ± 0.014	0.52 ± 0.07	28.8 ± 3.7	[16, 17]
sum	PP	0.57 ± 0.07	2.32 ± 1.27	41.1 ± 22.8	
1^-0^-	$\rho\pi$	12.7 ± 0.9	0.29 ± 0.07	0.23 ± 0.6	[30]
	$K^+\overline{K}^*(892)^- + c.c.$	5.0 ± 0.4	0.15 ± 0.08	0.3 ± 0.2	
	$K^0\overline{K}^*(892)^0 + c.c.$	4.2 ± 0.4	1.10 ± 0.20	2.6 ± 0.5	
	$\omega\pi^0$	0.42 ± 0.06	0.20 ± 0.06	4.8 ± 1.6	
1^-2^+	$\omega f_2(1270)$	4.3 ± 0.6	2.05 ± 0.56	4.8 ± 1.5	[15]
	ρa_2	10.9 ± 2.2	2.55 ± 0.87	2.3 ± 1.0	
	$K^*(892)^0\overline{K}_2^*(1430)^0 + c.c.$	6.7 ± 2.6	1.86 ± 0.54	2.8 ± 1.3	
	$\phi f_2'(1525)$	1.23 ± 0.21	0.44 ± 0.16	3.6 ± 1.4	
sum	VP& VT	45.45 ± 7.37	8.64 ± 2.54	1.90 ± 0.64	
$B\overline{B}$	$p\overline{p}$	2.12 ± 0.10	2.07 ± 0.31	9.8 ± 1.5	[29]
	$\Lambda\overline{\Lambda}$	1.30 ± 0.12	1.81 ± 0.34	13.9 ± 2.9	
	$\Sigma^0\overline{\Sigma}^0$	1.27 ± 0.17	1.2 ± 0.6	9.4 ± 4.9	
	$\Sigma(1385)^{\pm}\overline{\Sigma}(1385)^{\mp}$	1.03 ± 0.13	1.1 ± 0.4	10.7 ± 4.1	
	$\Xi\overline{\Xi}$	1.8 ± 0.4	1.88 ± 0.62†	10.4 ± 4.2	
	$\Delta^{++}\overline{\Delta}^{--}$	1.10 ± 0.29	1.28 ± 0.35	11.6 ± 4.4	
sum	$B\overline{B}$	8.62 ± 1.21	9.34 ± 2.62	10.8 ± 3.4	

Note: † simple normalization by $\Xi^-\overline{\Xi}^+ = (1/2)\Xi\overline{\Xi}$.

$\mathcal{B}(ggg) + \mathcal{B}(\gamma gg)$ by subtracting $\mathcal{B}(\gamma^*)$ and $\mathcal{B}(c\overline{c}X)$ from unity.

The calculated values of $\mathcal{B}(\gamma^*)$ and $\mathcal{B}(c\overline{c}X)$, together with the values used to calculate them are summarized in Table II. As regards to ψ', two final states $\gamma\eta(2S)$ and $h_c(1^1P_1)+X$ with faint branching fractions are neglected in our calculation. By deducting the contributions $\mathcal{B}(\gamma^*)$ and $\mathcal{B}(c\overline{c}X)$, we find that $\mathcal{B}(J/\psi \to ggg) + \mathcal{B}(J/\psi \to \gamma gg) = (73.5 \pm 0.6)\%$ and $\mathcal{B}(\psi' \to ggg) + \mathcal{B}(\psi' \to \gamma gg) = (19.1 \pm 2.5)\%$, then the ratio of them is

$$Q_g = \frac{\mathcal{B}(\psi' \to ggg + \gamma gg)}{\mathcal{B}(J/\psi \to ggg + \gamma gg)} = (26.0 \pm 3.5)\% \ . \tag{4}$$

The above estimation is consistent with the previous ones [32, 34]. The relation between the decay rates of ggg and γgg is readily calculated in pQCD to the first order as [35]

$$\frac{\Gamma(J/\psi \to \gamma gg)}{\Gamma(J/\psi \to ggg)} = \frac{16}{5}\frac{\alpha}{\alpha_s(m_c)}\left(1 - 2.9\frac{\alpha_s}{\pi}\right) .$$

Using $\alpha_s(m_c) = 0.28$, one can estimate the ratio to be 0.062. A similar relation can be deduced for the ψ' decays. So we obtain $\mathcal{B}(J/\psi \to ggg) \simeq (69.2 \pm 0.6)\%$ and $\mathcal{B}(\psi' \to ggg) \simeq (18.0 \pm 2.4)\%$, while the "26.0% ratio" in Eq. (4) stands well for both ggg and γgg. Although Q_g is considerably enhanced relative to Q_h in Eq. (1), it coincides with the ratio for the $K_S^0K_L^0$ decay mode between ψ' and J/ψ, which is

$$Q_{K_S^0K_L^0} = (28.8 \pm 3.7)\% \ , \tag{5}$$

according to the recent results from BES [16, 17]. The relation in Eq. (4) was discussed in the literature as the hadronic excess in ψ' decay [32, 34]. It implicates that while some modes are suppressed in ψ' decays, the dominant part of ψ' through ggg decays is enhanced relative to the 12% rule prediction in the light of J/ψ decays.

TABLE II: Experimental data on the branching fractions for J/ψ and ψ' decays through virtual photon and to lower mass charmonium states used in this analysis. Most of the data are taken from PDG [29], except for $\mathcal{B}(J/\psi, \psi' \to \gamma^* \to \text{hadrons})$, which are calculated by the product $R \cdot \mathcal{B}(J/\psi, \psi' \to \mu^+\mu^-)$, with $R = 2.28 \pm 0.04$ [33]. In estimating the errors of the sums, the correlations between the channels are considered.

Channel	$\mathcal{B}(J/\psi)$	$\mathcal{B}(\psi')$
$\gamma^* \to$ hadrons	(13.4±0.33)%	(1.66±0.18)%
e^+e^-	(5.93±0.10)%	$(7.55\pm0.31)\times10^{-3}$
$\mu^+\mu^-$	(5.88±0.10)%	$(7.3\pm0.8)\times10^{-3}$
$\tau^+\tau^-$		$(2.8\pm0.7)\times10^{-3}$
$\gamma^* \to X$	(25.22±0.43)%	(3.43±0.27)%
$\gamma\eta_c$	(1.3±0.4)%	$(2.8\pm0.6)\times10^{-3}$
$\pi^+\pi^-J/\psi$		(31.7±1.1)%
$\pi^0\pi^0J/\psi$		(18.8±1.2)%
$\eta J/\psi$		(3.16±0.22)%
π^0J/ψ		$(9.6\pm2.1)\times10^{-4}$
$\gamma\chi_{c0}$		(8.6±0.7)%
$\gamma\chi_{c1}$		(8.4±0.8)%
$\gamma\chi_{c2}$		(6.4±0.6)%
$c\overline{c}X$	(1.3±0.4)%	(77.4±2.5)%

III. THE CHARMLESS DECAYS OF ψ''

We define the enhancement or suppression factor as [8]

$$Q(f) \equiv \frac{\Gamma(\psi' \to f)}{\Gamma(J/\psi \to f)} \frac{\Gamma(J/\psi \to e^+e^-)}{\Gamma(\psi' \to e^+e^-)}. \quad (6)$$

In the $2S$-$1D$ mixing scheme, for any final state f, its partial width in ψ'' decay can be related to its partial width in J/ψ and ψ' decays by Eq.(3), with an unknown parameter ϕ which is the phase of η. This unknown phase constrains the predicted $\Gamma(\psi'' \to f)$ in a finite range. We calculate

$$R_\Gamma \equiv \Gamma(\psi'' \to f)/\Gamma(J/\psi \to f) \quad (7)$$

as a function of $Q(f)$ and plot it in Fig. 1. In the figure, the solid contour corresponds to the solution with $\phi = 0$; the dashed one corresponds to the solution with $\phi = 180°$; and the hatched area corresponds to the solutions with ϕ taking other non-zero values.

To make it clear, we discuss the final states in three situations: $Q(f) < 1$, $Q(f) > 1$, and $Q(f) = 1$.

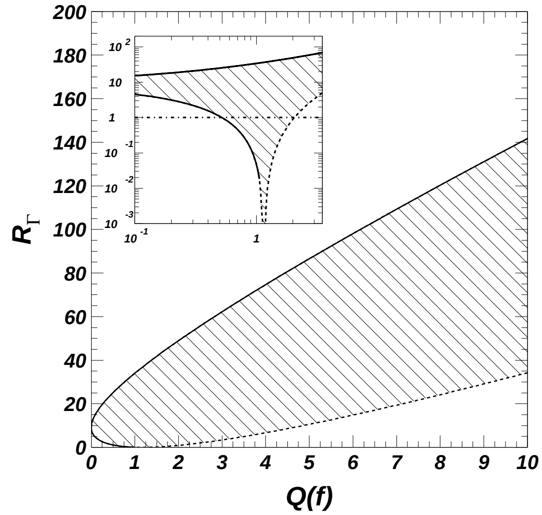

FIG. 1: R_Γ versus $Q(f)$. The solid contour corresponds to $\phi = 0$; the dashed contour corresponds to $\phi = 180°$; and the hatched area corresponds to ϕ having other non-zero values. The inset displays the variation of R_Γ in the vicinity of $Q(f) = 1$, where the dot-dashed line denotes $R_\Gamma = 1$.

A. Final states with $Q(f) < 1$

If $Q(f) < 1$, the decay $\psi' \to f$ is suppressed relative to $J/\psi \to f$. The extreme situation is $Q(f) \to 0$, corresponding to the absence of the mode f in ψ' decays. This is the case which was assumed for the $\rho\pi$ mode in the original work to solve the $\rho\pi$ puzzle by the S- and D-wave mixing [10]. If $Q(f) = 0$, the solution of the second equality of Eq. (3) simply yields $\eta = R_{2S}(0)\cos\theta/\sin\theta$ which cannot have a non-zero phase, and $R_\Gamma = 9.2$.

Generally, the suppression factor could be different from zero. Even the $\rho\pi$ and other strongly suppressed VP modes are found in ψ' decays recently by BESII [19] and CLEOc [22] with $Q(VP) \sim \mathcal{O}(10^{-2})$. In this case, there are two real and positive solutions of η as shown in Fig. 1 corresponding to the maximum and minimum of their possible partial widths in ψ'' decays. The solutions with η having a non-zero phase yield the values of R_Γ between the minimum and maximum limits.

For VT final states, which are measured to have $Q(VT) \approx 1/3$, with Eq. (3), we get $2.0 \le R_\Gamma \le 21.6$, as shown in Fig. 1, where the upper and lower limits correspond to two real and positive solutions of η, the range is due to the values of η with non-zero phases.

B. Final states with $Q(f) > 1$

If $Q(f) > 1$, the decay $\psi' \to f$ is enhanced relative to $J/\psi \to f$. The extreme situation is $Q(f) \to \infty$, corresponding to the complete absence of the final state f in J/ψ decays. For

$$Q(f) > \left| \frac{\cos\theta R_{2S}(0)}{\cos\theta R_{2S}(0) - \frac{5}{2\sqrt{2}m_c^2}\sin\theta R''_{1D}(0)} \right|^2 = 1.06,$$

there are two real solutions of η, one is positive and the other negative. From the first equality of Eq. (3), it is seen that the positive solution leads to the larger R_Γ. i.e. larger $\Gamma(\psi'' \to f)$ (the solid contour in Fig. 1) while the negative solution leads to the smaller $\Gamma(\psi'' \to f)$ (the dashed contour in Fig. 1).

For the known enhanced mode in ψ' decays like $K^0_S K^0_L$, $Q(K^0_S K^0_L) = 2.26$, we find $1.4 \le R_\Gamma \le 52.5$, which corresponds to the ψ'' decay partial width from 0.024 to 0.87 keV.

It should be noted that for finite $\Gamma(\psi' \to f)$, $Q(f) \to \infty$ means the diminishing of $\Gamma(J/\psi \to f)$, which gives rise to $R_\Gamma \to \infty$ according to the definition of R_Γ in Eq.(7). Under such circumstance, it is more intuitive to calculate

$$R'_\Gamma \equiv \Gamma(\psi'' \to f)/\Gamma(\psi' \to f).$$

Its variation as a function of $Q(f)$ is shown in Fig. 2. As $Q(f) \to \infty$, the solid and dashed contours converge into the same point ($R'_\Gamma \to 21$). In such case S-wave state does not decay to f, but D-wave does. Its partial width in ψ' and ψ'' decays comes solely from the contribution of η, or the D-wave matrix element.

C. Final states with $Q(f) = 1$

These final states observe the 12% rule in J/ψ and ψ' decays. In this case, apparently one solution is identical to the pure electromagnetic decays, with $\eta = 5R''_{1D}(0)/(2\sqrt{2}m_c^2)$ and $R_\Gamma = 0.048$. As in the leptonic decays, their partial widths in ψ'' decays are small relative to the partial widths in J/ψ decays. However, there are also other solutions with overwhelming contribution

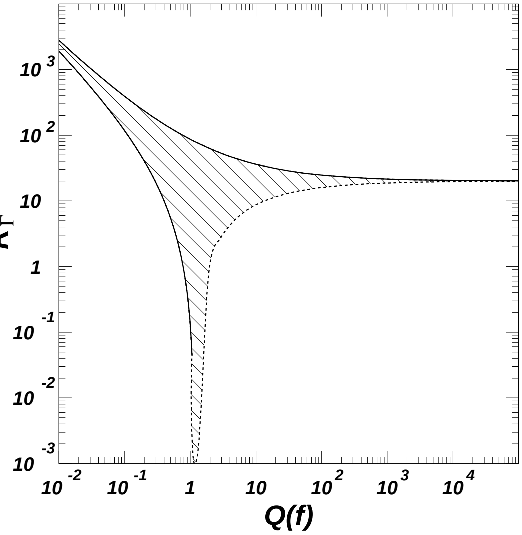

FIG. 2: R'_Γ versus $Q(f)$. The solid contour corresponds to $\phi = 0$; dashed contour corresponds to $\phi = 180°$; and the hatched area corresponds to ϕ having other non-zero values.

from η which lead to very large partial widths in ψ'' decays. This can be seen from Fig. 1 at the point with $Q(f) = 1$. We get the maximum of R_Γ of 34.0 which corresponds to another real and positive value of η. If η has a non-zero phase, then $0.048 < R_\Gamma < 34.0$.

D. Numerical results

From Fig. 1, we see that except in the range $0.52 < Q(f) < 2.06$ and a small range of the phase, R_Γ is always greater than 1. This range excludes virtually all known decay modes except $B\overline{B}$ which has $Q(B\overline{B}) \approx 1$. Even inside this range, there are other solutions by which R_Γ (or R'_Γ) is at $\mathcal{O}(10)$. It means in this scenario, contrary to the naïve guess, the charmless decay width of ψ'' is greater than that of J/ψ or ψ'. More surprising is that R_Γ could be as large as a few tens for the final states with $Q(f) > 1$. In general, the final states which are enhanced in ψ' decays possibly have a large combined partial width in ψ'' decays, especially if the phase of η is zero or very small.

There are reasons to assume that the phase ϕ which is between the matrix elements $\langle f|2S\rangle$ and $\langle f|1D\rangle$ should be small [11]. For the decay mode like $\rho\pi$, since there is almost complete cancellation between $\cos\theta R_{2S}(0)$ and $\eta\sin\theta$ so that $\langle\rho\pi|\psi'\rangle = \cos\theta R_{2S}(0) - \eta\sin\theta \approx 0$, the phase of η must be small. If this is to be extrapolated to all final states, the physics solution will follow the solid contour of Fig. 1. Another argument comes from the universal phase between the strong and electromagnetic amplitudes of the charmonium decays. It has been known that in the two-body decays of J/ψ, the phase between the strong and electromagnetic amplitudes is universally around 90° [31, 34]. Recently, it has been found that this phase is also consistent with the experimental data of ψ' and ψ'' decays [24, 36]. Since there is no extra phase between $2S$ and $1D$ matrix elements due to electromagnetic interaction, as in the calculations of the leptonic decay rates of ψ' and ψ'', the universal phase between the strong and electromagnetic interactions implies there is no extra phase between the two matrix elements due to the strong interaction too, *i.e.* $\phi \approx 0$. This conclusion means, for the modes which are enhanced in ψ' decays, their partial widths in ψ'' decay must be greater than those in J/ψ or ψ' decays by more than an order of magnitude.

In Sect. II, we estimate that $\mathcal{B}(J/\psi \to ggg) \simeq (69.2 \pm 0.6)\%$ while $\mathcal{B}(\psi' \to ggg) \simeq (18.0 \pm 2.4)\%$. Among the final states, we know that VP and VT modes have $Q(f) < 1$. For them,

$$\begin{aligned} \sum\mathcal{B}(J/\psi \to \mathrm{VP, VT}) &\approx 4.6\% , \\ \sum\mathcal{B}(\psi' \to \mathrm{VP, VT}) &\approx 8.6 \times 10^{-4} . \end{aligned}$$

Furthermore, there are final states with $Q(f) \approx 1$ (such as $B\overline{B}$), for them,

$$\begin{aligned} \sum\mathcal{B}(J/\psi \to B\overline{B}) &\approx 0.9\% , \\ \sum\mathcal{B}(\psi' \to B\overline{B}) &\approx 9.3 \times 10^{-4} . \end{aligned}$$

After subtracting the final states which are known to have $Q(f) < 1$ and $Q(f) \approx 1$, the remaining 63.8% of J/ψ decay with a total width of $\Gamma(J/\psi \to r.f.s.) \approx 58.1$ keV, and 17.8% of ψ' decays with a total width of $\Gamma(\psi' \to r.f.s.) \approx 50.1$ keV which are gluonic either has $Q(f) > 1$ or $Q(f)$ unknown. Here $r.f.s.$ stands for the remaining final states. On the average, these final states have

$$Q(r.f.s.) \approx 2.19 .$$

This is roughly comparable to $Q(K^0_S K^0_L) = 2.26$.

If there is no extra phase between $2S$ and $1D$ matrix elements as argued above, then R_Γ takes the maximum possible value with $R_\Gamma \approx 51.6$ for $Q(r.f.s.) \approx 2.19$. With this value, we find $\Gamma(\psi'' \to r.f.s.) = R_\Gamma \times \Gamma(J/\psi \to r.f.s.) \approx 3.0$ MeV for the partial width of these remaining final states in ψ'' decays, which is 13% of the total ψ'' width.

The above calculation takes the averaged $Q(f)$ for the final states with $Q(f) > 1$ and $Q(f)$ unknown, so it merely serves as a rough estimation. The exact value of the partial width should be the sum of the individual final states which in general have various $Q(f)$ values. At present a major impediment to do the accurate evaluation is the lack of experimental information. Nevertheless, if we take 13% as charmless decays (the calculations done channel by channel for VP, VT and $B\overline{B}$ modes in Table I give a summed maximum possible width of 93 keV in ψ'' decays, or 0.4% of the ψ'' total width) together with the charmonium transition contributions of 3% [8] (2.5% for radiative transition and 0.4% for $\pi\pi J/\psi$), we obtain a maximum of ψ'' non-$D\overline{D}$ decay branching fraction of 16% in the $2S$-$1D$ mixing scenario to be compared with 18% as summarized in Ref. [8].

IV. EXPERIMENTAL MEASUREMENTS

With the expected data of a few fb^{-1} at ψ'' from the running CLEOc and more from the future BES-III, it is of great importance to search for the charmless decays of ψ'' experimentally. These measurements test the $2S$-$1D$ mixing scenario on the explanation of the $\rho\pi$ puzzle in J/ψ and ψ' decays, and provide information on the relative phase between the $2S$ and $1D$ matrix elements, which is related to the charmonium decay dynamics.

Besides the theoretical interests, such measurements are important for the experiment itself. So far, in the fitting of the ψ'' resonance parameters with the scanned cross sections, it has been assumed that ψ'' decays completely into $D\overline{D}$ [37–40]. If the non-$D\overline{D}$ branching fraction is substantial, it must be considered in the fitting in order to get self-consistent results.

A. Inclusive method

A substantial non-$D\overline{D}$ decays of ψ'', including charmless decays and charmonium transitions, originally caught attention from the comparison between the cross sections of the inclusive hadrons and $D\overline{D}$ at the ψ'' peak. However, this is not unambiguous due to the poor statistics of the data samples and the complexity of the analysis. In addition, results from different experiments are consistent with each other only marginally.

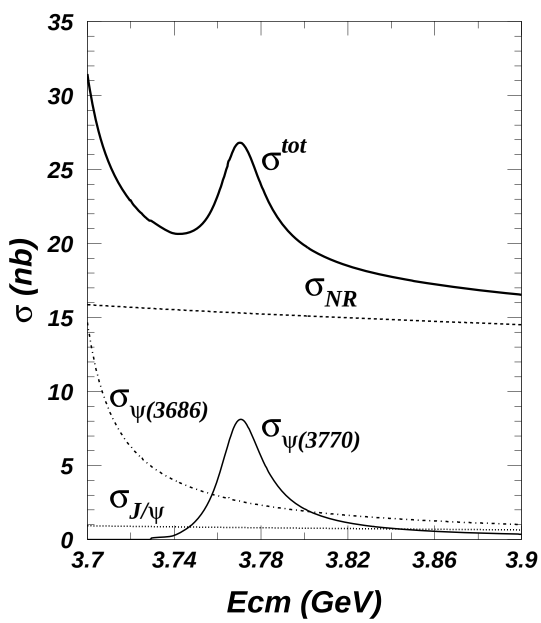

FIG. 3: The cross section $\sigma(e^+e^- \to \text{hadrons})$ in the vicinity of the ψ'' resonance calculated with parameters provided by PDG. The total cross section σ^{tot} is divided into four parts: the non-resonance σ_{NR}, the radiative tails of J/ψ ($\sigma_{J/\psi}$) and ψ' ($\sigma_{\psi'}$), and the ψ'' resonance ($\sigma_{\psi''}$).

Fig. 3 shows $\sigma(e^+e^- \to \text{hadrons})$ in the vicinity of the ψ'' resonance calculated with parameters by PDG [29]. The total cross section σ^{tot} can be expressed as

$$\sigma^{tot} = \sigma_{NR} + \sigma_{J/\psi} + \sigma_{\psi'} + \sigma_{\psi''}, \qquad (8)$$

which contains four parts: the non-resonance cross section σ_{NR}, the radiative tails of J/ψ ($\sigma_{J/\psi}$) and ψ' ($\sigma_{\psi'}$), and the ψ'' resonance ($\sigma_{\psi''}$). The non-resonance cross section is usually expressed in terms of R value and the μ pair cross section as $\sigma_{NR} = R\cdot\sigma(e^+e^- \to \mu^+\mu^-)$. The Breit-Wigner formula is adopted to depict resonances of J/ψ, ψ' and ψ'', where the total decay width of ψ'' is energy dependent:

$$\sigma_{\psi''}(E_{cm}) = \frac{12\pi\Gamma_{ee}\Gamma_{\psi''}(E_{cm})}{(E_{cm}^2 - M_{\psi''}^2)^2 + \Gamma_{\psi''}^2(E_{cm})M_{\psi''}^2},$$

with

$$\Gamma_{\psi''}(E_{cm}) = C_\Gamma\left[\frac{p_{D^0}^3}{1+(rp_{D^0})^2} + \frac{p_{D^\pm}^3}{1+(rp_{D^\pm})^2}\right], \qquad (9)$$

where p is the D^0 or $D^\pm$ momentum, r is the classical interaction radius, and C_Γ is defined as follows:

$$C_\Gamma \equiv \frac{\Gamma_{\psi''}(M_{\psi''})}{\left.\left[\frac{p_{D^0}^3}{1+(rp_{D^0})^2} + \frac{p_{D^\pm}^3}{1+(rp_{D^\pm})^2}\right]\right|_{E_{cm}=M_{\psi''}}}.$$

Here $\Gamma_{\psi''}(M_{\psi''})$ is the ψ'' total decay width given by PDG [29].

In previous analyses [37–40], the observed inclusive hadronic cross sections were fit with the theoretical one with the contributing terms in Eq. (8) to obtain the resonance parameters, which yield the ψ'' cross section at the peak. The comparison of this cross section with the $D\overline{D}$ cross section measured by tagging D mesons yields the non-$D\overline{D}$ decay branching fraction. An inconsistency exists in this procedure because in the fit to the total cross section, $D\overline{D}$ final states was assumed to saturate the ψ'' decays. Since light hadrons have much lower thresholds than $D\overline{D}$, a large non-$D\overline{D}$ branching fraction directly affects the shape of the resonance curve, and also indirectly through the energy-dependent width. Taking into account the non-$D\overline{D}$ decays, Eq. (9) should be revised by including another term, that is

$$\Gamma_{\psi''}(E_{cm}) = C'_\Gamma \times \left[\frac{p_{D^0}^3}{1+(rp_{D^0})^2} + \frac{p_{D^\pm}^3}{1+(rp_{D^\pm})^2} + C_{\text{non-}D\overline{D}}\right],$$

where $C_{\text{non-}D\overline{D}}$ is proportional to the part of the width from non-$D\overline{D}$ decays, and

$$C'_\Gamma \equiv \frac{\Gamma_{\psi''}(M_{\psi''})}{\left.\left[\frac{p_{D^0}^3}{1+(rp_{D^0})^2} + \frac{p_{D^\pm}^3}{1+(rp_{D^\pm})^2} + C_{\text{non-}D\overline{D}}\right]\right|_{E_{cm}=M_{\psi''}}}.$$

With the $C_{\text{non-}D\overline{D}}$ term in the expression for $\Gamma_{\psi''}$, the fitting of the resonance curve to extract the resonance parameters must be done together with the fitting of the $D\overline{D}$ cross section. In this procedure, the non-$D\overline{D}$ decay branching fraction is extracted together with the resonance parameters.

However, this method subtracts $D\overline{D}$ cross section from the total inclusive one to get the non-$D\overline{D}$ cross section which is only a fraction of the total inclusive cross section. It suffers from the large uncertainties of the measurements, so it is difficult to obtain a statistically significant result.

B. Exclusive method

The calculations in section III show that those final states which are suppressed in ψ' decays relative to J/ψ, and especially those ones enhanced in ψ' decays, will show up in ψ'' decays with maximum possible partial widths more than an order of magnitude greater than their widths in J/ψ or ψ' decays. So these exclusive charmless modes should be searched for in ψ'' decays. This provides direct test of the calculations based on the $2S$-$1D$ mixing scheme. Here we discuss three typical exclusive modes: VP mode which is suppressed in ψ' decays relative to J/ψ, PP mode which is enhanced, and of particular interest, the $B\overline{B}$ and $\phi f_0(980)$ modes which observe the 12% rule.

To measure the exclusive VP mode in ψ'' decays by e^+e^- experiments, the contribution from non-resonance virtual photon amplitude and its interference with the resonance must be treated with care. A recent study on the measurement of $\psi'' \to VP$ in e^+e^- experiments shows [24] that with the decay rate predicted by the S- and D-wave mixing, the interference between the three-gluon decay amplitude and the continuum one-photon amplitude leads to very small cross sections for some VP modes, e.g. $\rho\pi$ and $K^{*+}K^- + c.c.$, due to the destructive interference, but much larger cross sections for other VP modes, e.g. $K^{*0}\overline{K}^0 + c.c.$ due to the constructive interference. In another word, although the branching fractions of $\rho^0\pi^0$ and $K^{*0}\overline{K}^0$ differ by only a fraction due to SU(3) symmetry breaking [41], their production cross sections in e^+e^- collision differ by one to two orders of magnitude.

Among the PP modes, there is the $K^0_S K^0_L$ final state which decays only through strong interaction and does not couple to virtual photon [11, 41]. There is no complication of electromagnetic interaction and the interference between it and the resonance. So the observed $K^0_S K^0_L$ in e^+e^- experiment is completely from resonance decays. In the $2S$-$1D$ mixing scheme, with the BES recently reported $K^0_S K^0_L$ branching fractions in J/ψ [16] and ψ' [17] decays as inputs, it is estimated [11] $(1.2 \pm 0.7) \times 10^{-6} < \mathcal{B}(\psi'' \to K^0_S K^0_L) < (3.8 \pm 1.1) \times 10^{-5}$ [42]. If there is no extra phase between $\langle K^0_S K^0_L | 2^3S_1 \rangle$ and $\langle K^0_S K^0_L | 1^3D_1 \rangle$, then its branching fraction is at the upper bound. With $17.7\ \text{pb}^{-1}$ ψ'' data, BES has set an upper limit [43], which is still beyond the sensitivity for testing the above prediction. More precise determination of this branching fraction is expected from the analysis based on larger data samples of CLEOc and BES-III.

However, for other PP modes, or more generally other final states which are enhanced in ψ' decays, in e^+e^- experiments there is still the complication from the non-resonance virtual photon amplitude and its interference with the resonance.

Of particular interest are the final states with $Q(f) \approx 1$. These are the $B\overline{B}$ modes and the vector-scalar mode $\phi f_0(980)$ [44]. As discussed in subsection III C for $Q(f) = 1$, there are two real and positive solutions with $R_\Gamma = 0.048$ and $R_\Gamma = 34.0$. These two solutions are three orders of magnitude apart, their branching fractions are extremely sensitive to the relative phase between the $2S$ and $1D$ matrix elements. The $B\overline{B}$ branching fractions in J/ψ decays are at $\mathcal{O}(10^{-3})$, while $\phi f_0(980)$ is $(3.2 \pm 0.9) \times 10^{-4}$. If the physics solution of η is the larger one of the two real values, then the $B\overline{B}$ branching fraction in ψ'' decay would be at $\mathcal{O}(10^{-4})$ and $\phi f_0(980)$ would be 4.2×10^{-5}, which can be observed in the ψ'' data sample over $1\ \text{fb}^{-1}$.

V. SUMMARY

Based on the available experimental information of J/ψ and ψ' decays, we calculate the charmless decays of ψ'' by virtue of the S- and D-wave charmonia mixing scheme which was proposed to explain the large $\psi'' \to e^+e^-$ partial width and the $\rho\pi$ puzzle. We find that this leads to a possible large branching fraction, up to 13%, of the charmless final state in ψ'' decays. Although the calculation is semi-quantitative, it demonstrates that a large charmless branching fraction in ψ'' decays can well be explained.

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